

## A Fuzzy Inventory Model with Perishable and Aging Items

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### Abstract

A parametric multi-period inventory model for perishable items considered in this paper. Each item in the stock perishes in a given period of time with some uncertainty. A model derived for recursive unnormalized conditional distributions of  $\{\alpha_n\}$  given the information accumulated about the inventory level- surviving items processes.

**Key words:** optimal replenishment schedule, fuzzy Markov chain, fuzzy random variable, possibility space, fuzzy indicator function.

### I. Introduction

The limited lifetime of perishable products contribute greatly to the complexity of their management. Modeling perishable inventory is mainly stimulated by the economic impact of perishability. This model is inspired from a special type of time series models called First Order Integer-Valued Autoregressive process (INAR(1)). These models were introduced independently by hl-Osh and il-Zaid [1] and McKenzie [3] for modeling counting processes consisting of dependent variables. Demands are assumed to be random and the probability that an item perishes is not known with certainty. Expressions for various parameter estimates of the model are established and the problem of finding an optimal replenishment schedule is formulated as an optimal fuzzy stochastic control problem. Ishii et al. [5],[6] the authors focused on two types of customers (high and low priority), items of m different ages with different prices, and only a single-period decision horizon. Parlar [7] considers a perishable product that has two-periods of lifetime, where a fixed proportion of unmet demand for new items is fulfilled by unsold old items (and vice-versa). Goh et al. [4] considered a two-stage perishable inventory problem. Their model has random supply and separate demand modeled as a Poisson process.

#### 1.1 Assumption:

- Perishability rate is uncertain
- Demand rate are assumed to be random
- Shortages are not allowed.
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#### 1.2 Notations:

$X_n$  - the number items left at time epoch n (each item in the Shelf  
Perishes with possibility  $(1-\alpha)$ )

$V_n$  - number of new items arriving to the shelf with mean  $\mu_n$  and variance  $S_n^2$

$U_n$  - the replenishment quantity at time n

$\alpha_n$  -Homogeneous fuzzy Markov chain with finite state space S

Each item in the stock is assumed to perish in period n with Possibility  $(1-\alpha_n)$  where  $0 < \alpha_n < 1$ . The sequence  $\{\alpha_n\}$  is not known with certainty but it is known to belong to one of a finite set of states of a Fuzzy Markov chain. Let X be a fuzzy random variable. Then for any  $\alpha \in (0, 1)$ , define the operator o by:

$$\alpha o X = \sum_{j=1}^x Y_j \quad (1.1)$$

Where  $Y_j$  is the sequence of fuzzy random variable such that

$$\tilde{p}(Y_j = 1) = (1,1,1) - \tilde{p}(Y_j = 0) = \alpha \tag{1.2}$$

Then  $\{X_n\}$  is given by

$$X_n = \alpha \circ X_{n-1} + V_n \tag{1.3}$$

Where  $V_n$  is a sequence of uncorrelated fuzzy random variable with mean  $\mu_n$  and variance  $S_n^2$

Now let  $X_n$  represent the number of items at the end of period n in our inventory and consider the following extension of (1.3)

$$X_n = \alpha \circ X_{n-1} + V_n + U_n \tag{1.4}$$

$X_n$  takes only non – negative values, it is to be noted that negative values of  $X_n$  is interpreted as short age. Again  $V_n$  is a sequence of non negative fuzzy random variable with possibility function  $\Psi_n$  and is independent of  $Y_j^n$ . Further we assume that the sequence  $\{\alpha_n\}$  is a homogeneous fuzzy Markov chain with finite state space

S. For instance,  $\{\alpha_n\}$  expresses changes in  $\alpha$  caused by changes in the environment (temperature, humidity, air pressure, etc.) at different time epochs n.

The conditional probability distribution of  $\{\alpha_n\}$  given the information accumulated about the level of inventory  $\{X_n\}$  up to time n, we have also estimate the transition probabilities of the Markov chain  $\{\alpha_n\}$ . This model is closely related to random life model. However, due to the complexity inherent to the analysis of models where units have a random lifetime, very little progress has been made on that front, apart from suggesting equivalent deterministic or using queuing models for their analysis.

## II. Conditional fuzzy Possibility Distribution of $\{\alpha_n\}$ :

Let  $I_1, \dots, I_M$  be a partition of the interval (0,1). Also, let  $s_1$  be any point in  $I_1$ ,  $s_2$ , be any point in  $I_2, \dots, s_M$  be any point in  $I_M$ , and  $S = \{s_1, \dots, s_M\}$ .

Suppose that the sequence  $\{\alpha_n\}$  is a homogeneous fuzzy Markov chain with state space S describing the evolution of the parameter  $\tilde{\alpha}_n$  of the inventory model. All random variables are initially defined on a possibility space  $(\Gamma, \psi, \sigma)$ . For  $1 \leq i, j \leq M$ , write

$$\begin{aligned} \tilde{p}_{ij} &= \sigma((\alpha_n \in I_i | \alpha_{n-1} \in I_j)) \\ &= \sigma((\alpha_n = s_i | \alpha_{n-1} = s_j)) \end{aligned} \tag{2.1}$$

$$\tilde{P} = \tilde{p}_{ij}, \sum \tilde{p}_{ij} = (1,1,1)$$

Recall that  $X_n$  represents the inventory level at time n. Now let

$$A_n = \sigma\{\alpha_n, K \leq n\} \tag{2.2}$$

$$B_n = \sigma\{X_k, Y_i^k, K \leq n, i \geq 1\} \tag{2.3}$$

$$G_n = \sigma\{X_k, Y_i^k, \alpha_n, K \leq n, i \geq 1\} \tag{2.4}$$

be the complete filtrations generated by the parameter process  $\{\alpha_n\}$  the inventory level-surviving items processes  $\{X_n, Y_i^n\}$ , and the inventory level-surviving items and the parameter processes respectively.

Note that  $A_n$  and  $B_n \subset G_n$  for  $n-1$ , Following the techniques of Elliot, Aggoun, and Moore [2], we introduce a new possibility measure  $\tilde{P}$  under which  $\{\alpha_n\}$  is a sequence of i.i.d, fuzzy random variables uniformly distributed on the set  $S = \{s_1, \dots, s_M\}$  and  $\{X_n\}$  is a sequence of independent fuzzy random variables with possibility distributions  $\Psi_n$  independent of  $\{\alpha_n\}$ . For this, define the factors:

$$\lambda_0 = 1$$

$$\lambda_n = \prod_{i=1}^M (M\alpha_n^i)^{-I(\alpha_n = s_i)} \left\{ \frac{\Psi_n(X_n - \alpha_n \circ X_{n-1} - U_n)}{\Psi_n} \right\}^{-1} \tag{2.5}$$

Where  $I(\alpha = S_i)$  is the fuzzy indicator function of the event  $(\alpha_n = S_i)$  and

$$\alpha_n = \sum_{j=1}^n I(\alpha_{n-1} = s_i) \tilde{p}_{ij} \quad (2.6)$$

Note that  $\{\alpha_n^i\}$  is  $A_{n-1}$  adapted and

$$E[I(\alpha_n = S_i | A_{n-1})] = \alpha_n^i,$$

so that,  $I(\alpha_n = S_i) - \alpha_n^i = \Delta_n^i$

$$A_n = \prod_{k=0}^n \lambda_k \quad (2.7)$$

Then  $\{A_n\}$  is a  $G_n$ -martingale such that  $E[\{A_n\}] = 1$ . We can define a new probability measure  $\tilde{P}$  on  $(\Gamma, \mathcal{V}_{n=1}^\infty G_n)$  by setting

$$\frac{d\tilde{P}}{d\sigma} \Big|_{G_n} = A_n \quad (2.8)$$

The existence of  $\tilde{P}$  on  $(\Gamma, \mathcal{V}_{n=1}^\infty G_n)$  is due to Kolmogorov's extension theorem. Then under  $\tilde{P}$ ,  $\{X_n\}$  is a sequence of independent random variables with probability distributions  $\Psi_n$  and  $\{\alpha_n\}$  is a sequence of i.i.d, random variables uniformly distributed over S. Write

$$\tilde{p}(S_i) = \sigma(\alpha_{n-1} = s_i | B_n) = E\{I(\alpha_n = s_i) | B_n\}, \quad (2.9)$$

And

$$\tilde{q}_n(s_i) = E\{I(\alpha_n = s_i) A_n^{-1} | B_n\} \quad (2.10)$$

Here  $E$  denotes the expectation under  $\tilde{P}$  and  $A_n$ . Hence

$$\tilde{q}_n(s_i) = \frac{\Psi_1(X_1 - s_i \circ X_0 - U_1)}{\Psi_1(X_1)} \tilde{p}_{i, \alpha_0} \quad (2.11)$$

$$\begin{aligned} \tilde{P}(\alpha_1 = s_i | B_1) &= \tilde{p}_1(s_i) = \frac{\tilde{q}_n(s_i)}{\sum_{k=1}^M \tilde{q}_k(s_i)} \\ &= \frac{\Psi_1(X_1 - s_i \circ X_0 - U_1) \tilde{p}_{i, \alpha_0}}{\sum_{k=1}^M \Psi_1(X_1 - s_k \circ X_0 - U_1) \tilde{p}_{k, \alpha_0}}. \end{aligned} \quad (2.1)$$

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